APPENDIX B: REFERENCE EQUATIONS USED IN SECTION 5

B.1 BPSK Probability of Error

The following equations are from [1] and are used for calculating bit error probabilities P_b, where R_d is the symbol energy-to-noise ratio.

● BPSK [eq. 5-47, ref. 1]

$$P_b = \frac{1}{2} erfc(\sqrt{R_d})$$

● DEBPSK [eq. 5-114, ref. 1]

$$P_b = erfc(\sqrt{R_d}) - \frac{1}{2} erfc^2(\sqrt{R_d})$$

● DBPSK [eq. 5-125, ref. 1]

$$P_b = \frac{1}{2}\exp(-R_d)$$

B.2 QPSK Probability of Error

The following CV code equation is from [2]. For the (1, 2, 7) CV code, $N = 20$, $d_{free} = 10$, and β is an array having values 36, 211, 1,404, 11,633, 76,628, and 469,991 for elements 10, 12, 14, 16, 18, and 20 respectively.

$$P_b < \sum_{d=d\ free}^{N} \beta_d P_2(d)$$

● $P_2(d)$ for QPSK and DEQPSK

$$P_2(d) = \frac{1}{2} erfc(\sqrt{\frac{R_d d}{2}}) = Q(\sqrt{R_d d})$$

● $P_2(d)$ for DQPSK

$$P_2(d) = \frac{2}{3}[erfc(\sqrt{R_d d}\ \sin(\frac{\pi}{4\sqrt{2}}))] = \frac{4}{3}[Q(\sqrt{2R_d d}\ \sin(\frac{\pi}{4\sqrt{2}}))]$$

B.3 Carrier Recovery

The carrier recovery equations are from [3]. In the following equations, R is the input signal-to-noise ratio, B_N is the narrowband filter bandwidth, T is the symbol period, σ_φ is the rms phase jitter in radians, φ is the instantaneous phase, $\hat{\varphi}$ is the average phase, and $E\{\bullet\}$ is the expectation operator.

$$\sigma_\varphi^2 = E\{\varphi - \hat{\varphi}\}^2$$

- BPSK

$$\sigma_\varphi^2 = B_N T [\frac{1}{2R} + \frac{1}{4R^2}]$$

- QPSK

$$\sigma_\varphi^2 = B_N T [0.1125 + 1.4625\frac{1}{R} + 24.469\frac{1}{R^2} + 21.094\frac{1}{R^3} + 2.531\frac{1}{R^4}]$$

B.4 Symbol Timing Recovery

The symbol timing recovery equations are from [4]. In the following equations, T is the symbol period, τ is the instantaneous delay, $\hat{\tau}$ is the average delay, ξ is the relative delay error, $\pi\sigma_\xi$ is the rms timing jitter in radians, R_s is the symbol rate, and B_N is the narrowband filter bandwidth. The relative delay error is expressed as:

$$\xi = \frac{(\tau - \hat{\tau})}{T}$$

and the variance can be expressed as

$$(\pi\sigma_\xi)^2 = \frac{\pi^2 B_N N_0 (1 + \frac{N_0}{2E_s})}{4E_s R_s}$$

B.5 Reed Solomon Code

The RS code equations were derived from results in [2]. N is the number of symbols in the codeword, p is the probability of input bit error, m is the number of bits per symbol, t is the number of correctable symbol errors, and P_b is the output bit error rate.

$$P_b = \sum_{i=t+1}^{N} \binom{N}{i} \frac{i}{2N} (mp)^i (1 - mp)^{N-i}$$

B.6 References

[1] W.C. Lindsey and M.K. Simon, *Telecommunication System Engineering,* Englewood Cliffs, NJ: Prentice Hall, 1973.

[2] J.G. Proakis, *Digital Communications 3rd Edition,* New York, NY: McGraw Hill, pp. 464-466 and 470-511.

[3] L.E. Franks, "Carrier and bit synchronization in data communications - A tutorial review," *IEEE Transactions on Communications,* vol. COM-28, no. 8, pp. 1107-21, 1980.

[4] L.C. Palmer, S.A Rhodes, and S.H. Lebowitz, "Synchronization for QPSK transmission via communication satellites," *IEEE Trans. Comm.,* vol. COM-28, no. 10, pp. 1302-1314, 1980.